

DSC-1A
BS:104

DIFFERENTIAL CALCULUS

Theory: 4 credits and Practicals: 1 credit
Theory: 4 hours/week and Practicals: 2 hours/week

Objective: the course is aimed at exposing the students to some basic notions in differential calculus.

Outcome: by the time students complete the course they realize wide ranging applications of the subject.

Unit – I

Successive differentiation:

Higher order derivatives, Calculation of the n th derivative, Some standard results, Determination of n th derivative of rational functions, The n th derivatives of the products of the powers of sines and cosines, Leibnitz's theorem, The n th derivative of the product of two functions.

Expansion of Functions:

Maclaurin's theorem, Taylor's theorem.

Mean Value Theorems:

Rolle's theorem, Lagrange's mean value theorem, Meaning of the sign of derivative, Graphs of hyperbolic functions, Cauchy's mean value theorem, Higher derivatives, Formal expansions of functions.

Unit – II

Indeterminate Forms:

Indeterminate forms, The indeterminate form $0/0$, The indeterminate form ∞/∞ , The indeterminate form $0 \cdot \infty$, The indeterminate form $\infty - \infty$, The indeterminate forms 0^0 , 1^∞ , ∞^0 .

Curvature and Evolutes:

Introduction, Definition of curvature, Length of arc as a function, Derivative of arc, Radius of curvature-cartesian equations, Newtonian method, Centre of curvature, Chord of curvature, Evolutes and involutes, Properties of the evolute.

Unit – III

Partial Differentiation – Homogeneous Functions – Total Derivative:

Introduction, Functions of two variables, Neighbourhood of a point (a, b) , Continuity of a Function of two variables, continuity at a point, Limit of a function of two variables, Partial derivatives, Geometrical representation of a function of two variables, Homogeneous functions, Theorem on total differentials; composite functions; differentiation of composite functions; implicit functions.

Unit – IV

Maxima and Minima:

Maxima and minima of function of two variables, Lagrange's method of undetermined multipliers.

Asymptotes:

Definition, Determination of asymptotes, Working rules of determining asymptotes, Asymptotes by inspection, Intersection of a curve and its asymptotes, Asymptotes by expansion, Position of a curve with respect to an asymptote, Asymptotes in polar co-ordinates.

Envelopes:

One parameter family of curves, Consider the family of straight lines, Definition, Determination of envelope, Theorem, To prove that, in general, the envelope of a family of curves touches each member of the family, If A, B, C are functions of x and y and m is a parameter then the envelope of $Am^2+Bm+C = 0$ is $B^2 = 4AC$, Two parameters connected by a relation, When the equation to a family of curves is not given, but the law is given in accordance with which any member of the family can be determined, Envelopes of polar curves, Envelopes of normals(Evolutes).

Text: Shanti Narayan and Mittal, Differential Calculus

References: William Anthony Granville, Percy F Smith and William Raymond Longley, Elements of the Differential and integral calculus

Joseph Edwards, Differential calculus for beginners

Smith and Minton, Calculus

Elis Pine, How to Enjoy Calculus

Hari Kishan, Differential Calculus

2.1.1 Practicals Question Bank

Differential Calculus

Unit-I

1. If $u = \tan^{-1} x$ prove that

$$(1 + x^2) \frac{d^2u}{dx^2} + 2x \frac{du}{dx} = 0$$

and hence determine the values of the derivatives of u when $x = 0$.

2. If $y = \sin(m \sin^{-1} x)$ show that

$$(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$$

and find $y_n(0)$

3. If U_n denotes the n th derivative of $\frac{Lx+M}{x^2-2Bx+C}$, prove

$$\frac{x^2 - 2Bx + C}{(n+1)(n+2)} U_{n+2} + \frac{2(x-B)}{n+1} U_{n+1} + U_n = 0$$

4. If $y = x^2 e^x$, then

$$\frac{d^n y}{dx^n} = \frac{1}{2} n(n-1) \frac{d^2 y}{dx^2} - n(n-2) \frac{dy}{dx} + \frac{1}{2} (n-1)(n-2)y.$$

5. Determine the intervals in which the function

$$(x^4 + 6x^3 + 17x^2 + 32x + 32)e^{-x}$$

is increasing or decreasing.

6. Separate the intervals in which the function

$$\frac{(x^2 + x + 1)}{(x^2 - x + 1)}$$

is increasing or decreasing.

7. Show that if $x > 0$,

$$(i) \quad x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}.$$

$$(ii) \quad x - \frac{x^2}{2} + \frac{x^3}{3(1+x)} < \log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}.$$

8. Prove that

$$e^{ax} \sin bx = bx + abx^2 + \frac{3a^2b - b^3}{3!} x^3 + \dots + \frac{(a^2 + b^2)^{\frac{1}{2}n}}{n!} x^n \sin(n \tan^{-1} \frac{b}{a}) + \dots$$

9. Show that

$$\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 \dots\dots\dots$$

10. Show that

$$e^{m \tan^{-1} x} = 1 + mx + \frac{m^2}{2!}x^2 + \frac{m(m^2 - 2)}{3!}x^3 + \frac{m^2(m^2 - 8)}{4!}x^4 + \dots$$

Unit-II

11. Find the radius of curvature at any point on the curves

(i) $y = c \cosh\left(\frac{x}{c}\right)$. (Catenary)

(ii) $x = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$.

(iii) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. (Astroid)

(iv) $x = \frac{(a \cos t)}{t}$, $y = \frac{(a \sin t)}{t}$.

12. Show that for the curve

$$x = a \cos \theta(1 + \sin \theta), y = a \sin \theta(1 + \cos \theta),$$

the radius of curvature is a at the point for which the value of the parameter is $\frac{-\pi}{4}$.

13. Prove that the radius of curvature at the point $(-2a, 2a)$ on the curve $x^2y = a(x^2 + y^2)$ is $-2a$.

14. Show that the radii of curvature of the curve

$$x = ae^{\theta}(\sin \theta - \cos \theta), y = ae^{\theta}(\sin \theta + \cos \theta)$$

and its evolute at corresponding points are equal.

15. Show that the whole length of the evolute of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is $4\left(\frac{a^2}{b} - \frac{b^2}{a}\right)$.

16. Show that the whole length of the evolute of the astroid

$$x = a \cos^3 \theta, y = a \sin^3 \theta$$

is $12a$

17. Evaluate the following:

(i) $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$

(ii) $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$

(iii) $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)}$

(iv) $\lim_{x \rightarrow 0} \left\{ \frac{1}{x} - \frac{1}{x^2} \log(1+x) \right\}$

18. If the limit of

$$\frac{\sin 2x + a \sin x}{x^8}$$

as x tends to zero, be finite, find the value of a and the limit.

19. Determine the limits of the following functions:

(i) $x \log(\tan x), (x \rightarrow 0)$

(ii) $x \tan(\pi/2 - x), (x \rightarrow 0)$

(iii) $(a - x) \tan(\pi x/2a), (x \rightarrow 0)$

20. Determine the limits of the following functions:

(i) $\frac{e^x - e^{-x} - x}{x^2 \sin x}, (x \rightarrow 0)$

(ii) $\frac{\log x}{x^3}, (x \rightarrow \infty)$

(iii) $\frac{1+x \cos x - \cosh x - \log(1+x)}{\tan x - x}, (x \rightarrow 0)$

(iv) $\frac{\log(1+x) \log(1-x) - \log(1-x^2)}{x^4}, (x \rightarrow 0)$

Unit-III

21. If $z = xyf(x/y)$ then show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z$$

22. If $z(x+y) = x^2 + y^2$ then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 = 4 \left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)$$

23. If $z = 3xy - y^3 + (y^2 - 2x)^{\frac{3}{2}}$, verify that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} \quad \text{and} \quad \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \left(\frac{\partial^2 z}{\partial x \partial y} \right)^2$$

24. If $z = f(x+ay) + \varphi(x-ay)$, prove that

$$\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}.$$

25. If $u = \tan^{-1} \left(\frac{x^3+y^3}{x-y} \right)$, find

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

26. If $f(x, y) = 0, \varphi(y, z) = 0$, show that

$$\frac{\partial f}{\partial y} \cdot \frac{\partial \varphi}{\partial z} \cdot \frac{dz}{dx} = \frac{\partial f}{\partial x} \cdot \frac{\partial \varphi}{\partial y}.$$

27. If $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$, show that

$$\frac{d^2 y}{dx^2} = \frac{a}{(1-x^2)^{\frac{3}{2}}}.$$

28. Given that $f(x, y) \equiv x^3 + y^3 - 3axy = 0$, show that

$$\frac{d^2 y}{dx^2} \cdot \frac{d^2 x}{dy^2} = \frac{4a^6}{xy(xy - 2a^2)^3}.$$

29. If u and v are functions of x and y defined by

$$x = u + e^{-v} \sin u, y = v + e^{-v} \cos u,$$

prove that

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}.$$

30. If $H = f(y - z, z - x, x - y)$; prove that,

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0.$$

Unit-IV

31. Find the minimum value of $x^2 + y^2 + z^2$ when

(i) $x + y + z = 3a$

(ii) $xy + yz + zx = 3a^2$

(iii) $xyz = a^3$

32. Find the extreme value of xy when

$$x^2 + xy + y^2 = a^2.$$

33. In a plane triangle, find the maximum value of

$$\cos A \cos B \cos C.$$

34. Find the envelope of the family of semi-cubical parabolas

$$y^2 - (x + a)^3 = 0.$$

35. Find the envelope of the family of ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where the parameters a, b are connected by the relation

$$a + b = c;$$

c , being a constant.

36. Show that the envelope of a circle whose centre lies on the parabola $y^2 = 4ax$ and which passes through its vertex is the cissoid

$$y^2(2a + x) + x^3 = 0.$$

37. Find the envelope of the family of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a, b are connected by the relation

(i) $a + b = c$.

(ii) $a^2 + b^2 = c^2$.

(iii) $ab = c^2$.

c is a constant.

38. Find the asymptotes of

$$x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0.$$

39. Find the asymptotes of

$$y^3 + x^3 + y^2 + x^2 - x + 1 = 0.$$

40. Find the asymptotes of the following curves

(i) $xy(x + y) = a(x^2 - a^2)$

(ii) $y^3 - x^3 + y^2 + x^2 + y - x + 1 = 0$.



2.2 Differential Equations

DSC-1B

BS:204

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The main aim of this course is to introduce the students to the techniques of solving differential equations and to train to apply their skills in solving some of the problems of engineering and science.

Outcome: After learning the course the students will be equipped with the various tools to solve few types differential equations that arise in several branches of science.

Unit- I

Differential Equations of first order and first degree: Exact differential equations - Integrating Factors - Change in variables - Total Differential Equations - Simultaneous Total Differential Equations - Equations of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. Differential Equations first order but not of first degree: Equations Solvable for y - Equations Solvable for x - Equations that do not contain x (or y)- Clairaut's equation.

Unit- II

Higher order linear differential equations: Solution of homogeneous linear differential equations with constant coefficients - Solution of non-homogeneous differential equations $P(D)y = Q(x)$ with constant coefficients by means of polynomial operators when $Q(x) = be^{ax}, b \sin ax/b \cos ax, bx^k, Ve^{ax}$.

Unit- III

Method of undetermined coefficients - Method of variation of parameters - Linear differential equations with non constant coefficients - The Cauchy - Euler Equation.

Unit- IV

Partial Differential equations- Formation and solution- Equations easily integrable - Linear equations of first order - Non linear equations of first order - Charpit's method - Homogeneous linear partial differential equations with constant coefficient - Non homogeneous linear partial differential equations - Separation of variables.

Text:

- Zafar Ahsan, *Differential Equations and Their Applications*

References:

- Frank Ayres Jr, *Theory and Problems of Differential Equations*.
 - Ford, L.R ; *Differential Equations*.
 - Daniel Murray, *Differential Equations*.
 - S. Balachandra Rao, *Differential Equations with Applications and Programs*.
 - Stuart P Hastings, J Bryce McLead; *Classical Methods in Ordinary Differential Equations*.
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2.2.1 Practicals Question Bank

Differential Equations

Unit-I

Solve the following differential equations:

1. $y' = \sin(x + y) + \cos(x + y)$
2. $x dy - y dx = a(x^2 + y^2) dy$
3. $x^2 y dx - (x^3 + y^3) dy = 0$
4. $(y + z) dx + (x + z) dy + (x + y) dz = 0$
5. $y \sin 2x dx - (1 + y^2 + \cos^2 x) dy = 0$
6. $y + px = p^2 x^4$
7. $yp^2 + (x - y)p - x = 0$
8. $\frac{dx}{y-zx} = \frac{dy}{yz+x} = \frac{dz}{(x^2+y^2)}$
9. $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$
10. Use the transformation $x^2 = u$ and $y^2 = v$ to solve the equation $axy p^2 + (x^2 - ay^2 - b)p - xy = 0$

Unit-II

Solve the following differential equations:

11. $D^2 y + (a + b) Dy + aby = 0$
12. $D^3 y - D^2 y - Dy - 2y = 0$
13. $D^3 y + Dy = x^2 + 2x$
14. $y'' + 3y' + 2y = 2(e^{-2x} + x^2)$
15. $y^{(5)} + 2y''' + y' = 2x + \sin x + \cos x$
16. $(D^2 + 1)(D^2 + 4)y = \cos \frac{x}{2} \cos \frac{3x}{2}$
17. $(D^2 + 1)y = \cos x + xe^{2x} + e^x \sin x$
18. $y'' + 3y' + 2y = 12e^x$
19. $y'' - y = \cos x$
20. $4y''' - 5y' = x^2 e^x$

Unit-III

Solve the following differential equations:

21. $y'' + 3y' + 2y = xe^x$
22. $y'' + 3y' + 2y = \sin x$
23. $y'' + y' + y = x^2$
24. $y'' + 2y' + y = x^2e^{-x}$
25. $x^2y'' - xy' + y = 2 \log x$
26. $x^4y''' + 2x^3y'' - x^2y' + xy = 1$
27. $x^2y'' - xy' + 2y = x \log x$
28. $x^2y'' - xy' + 2y = x$

Use the reduction of order method to solve the following homogeneous equation whose one of the solution is given:

29. $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0, y_1 = x$
30. $(2x^2 + 1)y'' - 4xy' + 4y = 0, y_1 = x$

Unit-IV

31. Form the partial differential equation, by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$.
32. Find the differential equation of the family of all planes whose members are all at a constant distance r from the origin.
33. Form the differential equation by eliminating arbitrary function F from $F(x^2 + y^2, z - xy) = 0$.

Solve the following differential equations:

34. $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$
35. $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$
36. $(p^2 - q^2)z = x - y$
37. $z = px + qy + p^2q^2$
38. $z^2 = pqxy$
39. $z^2(p^2 + q^2) = x^2 + y^2$
40. $r + s - 6t = \cos(2x + y)$

Theory: 4 credits and Practical 1 credit
Theory: 4 hours/week and Practicals : 2 hours/ week

Objective : The course is aimed at exposing the students to the foundations of analysis which will be useful in understanding various physical phenomena.

Outcome: After the completion of the course students will be in a position to appreciate beauty and applicability of the course.

Unit- I

Sequences- Limits of sequences- A Discussion about Proofs- Limit Theorems for Sequences – Monotone Sequences and Cauchy Sequences

Unit- II

Subsequences- Lim sup's and Lim inf's Series- Alternating Series and Integrals Tests.
Continuity : Continuous functions- Properties of Continuous functions.

Unit – III

Sequence and Series of Functions: Power Series- Uniform Convergence – More on Uniform Convergence- Differentiation and Integration of Power Series (Theorems in this section without Proofs)

Unit – IV

Integration : The Riemann Integral- Properties of Riemann Integral- Fundamental Theorem of Calculus.

Text : Kenneth A Ross, Elementary Analysis- The Theory of Calculus

References :

William F.Trench: Introduction to Real Analysis

Lee Larson: Introduction to Real Analysis

Shanti Narayan and Mittal: Mathematical Analysis

Brian S. Thomson, Judith B. Bruckner, Andrew M. Bruckner: Elementary Real Analysis

Sudhir R. Ghorpade Balmohan V. Limaye: A Course in Calculus and Real Analysis

2.5.1 Practicals Question Bank

Real Analysis

Unit-I

1. For each sequence below, determine whether it converges and, if it converges, give its limit. No proofs are required.

(a) $a_n = \frac{n}{n+1}$

(b) $b_n = \frac{n^2+3}{n^2-3}$

(c) $c_n = 2^{-n}$

(d) $t_n = 1 + \frac{2}{n}$

(e) $x_n = 73 + (-1)^n$

(f) $s_n = (2)^{\frac{1}{n}}$

2. Determine the limits of the following sequences, and then prove your claims.

(a) $a_n = \frac{n}{n^2+1}$

(b) $b_n = \frac{7n-19}{3n+7}$

(c) $c_n = \frac{4n+3}{7n-5}$

(d) $d_n = \frac{2n+4}{5n+2}$

(e) $s_n = \frac{1}{n} \sin n$

3. Suppose $\lim a_n = a$, $\lim b_n = b$, and $s_n = \frac{a_n^3+4a_n}{b_n^2+1}$. Prove $\lim s_n = \frac{a^3+4a}{b^2+1}$ carefully, using the limit theorems.

4. Let $x_1 = 1$ and $x_{n+1} = 3x_n^2$ for $n \geq 1$.

(a) Show if $a = \lim x_n$, then $a = \frac{1}{3}$ or $a = 0$.

(b) Does $\lim x_n$ exist? Explain.

(c) Discuss the apparent contradiction between parts (a) and (b).

5. Which of the following sequences are increasing? decreasing? bounded?

(a) $\frac{1}{n}$

(b) $\frac{(-1)^n}{n^2}$

(c) n^5

(d) $\sin(\frac{n\pi}{7})$

(e) $(-2)^n$

(f) $\frac{n}{3^n}$

6. Let (s_n) be a sequence such that $|s_{n+1} - s_n| < 2^{-n}$ for all $n \in \mathbb{N}$. Prove (s_n) is a Cauchy sequence and hence a convergent sequence.

7. Let (s_n) be an increasing sequence of positive numbers and define $\sigma_n = \frac{1}{n}(s_1 + s_2 + \dots + s_n)$. Prove (σ_n) is an increasing sequence.

8. Let $t_1 = 1$ and $t_{n+1} = [1 - \frac{1}{4n^2}].t_n$ for $n \geq 1$.

(a) Show $\lim t_n$ exists.

(b) What do you think $\lim t_n$ is?

- (e) $\limsup s_n + \limsup t_n$, (f) $\liminf(s_n t_n)$,
 (g) $\limsup(s_n t_n)$.

15. Determine which of the following series converge. Justify your answers.

- (a) $\sum \frac{n^4}{2^n}$ (b) $\sum \frac{2^n}{n!}$
 (c) $\sum \frac{n^2}{3^n}$ (d) $\sum \frac{n!}{n^4+3}$
 (e) $\sum \frac{\cos^2 n}{n^2}$ (f) $\sum_{n=2}^{\infty} \frac{1}{\log n}$

16. Prove that if $\sum a_n$ is a convergent series of nonnegative numbers and $p > 1$, then $\sum a_n^p$ converges.

17. Show that if $\sum a_n$ and $\sum b_n$ are convergent series of nonnegative numbers, then $\sum \sqrt{a_n b_n}$ converges.

Hint: Show $\sqrt{a_n b_n} \leq a_n + b_n$ for all n .

18. We have seen that it is often a lot harder to find the value of an infinite sum than to show it exists. Here are some sums that can be handled.

- (a) Calculate $\sum_{n=1}^{\infty} (\frac{2}{3})^n$ and $\sum_{n=1}^{\infty} (-\frac{2}{3})^n$.
 (b) Prove $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. Hint: Note that $\sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^n [\frac{1}{k} - \frac{1}{k+1}]$.
 (c) Prove $\sum_{n=1}^{\infty} \frac{n-1}{2^{n+1}} = \frac{1}{2}$. Hint: Note $\frac{k-1}{2^{k+1}} = \frac{k}{2^k} - \frac{k+1}{2^{k+1}}$.
 (d) Use (c) to calculate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

19. Determine which of the following series converge. Justify your answers.

- (a) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n \log n}}$ (b) $\sum_{n=2}^{\infty} \frac{\log n}{n}$
 (c) $\sum_{n=4}^{\infty} \frac{1}{n(\log n)(\log \log n)}$ (d) $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$

20. Show $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if and only if $p > 1$.

UNIT-III

21. For each of the following power series, find the radius of convergence and determine the exact interval of convergence.

- (a) $\sum n^2 x^n$ (b) $\sum (\frac{x}{n})^n$
 (c) $\sum (\frac{2^n}{n^2}) x^n$ (d) $\sum (\frac{n^3}{3^n}) x^n$
 (e) $\sum (\frac{2^n}{n!}) x^n$ (f) $\sum (\frac{1}{(n+1)^{2 \cdot 2^n}}) x^n$

(g) $\sum (\frac{3^n}{n \cdot 4^n}) x^n$

(h) $\sum (\frac{(-1)^n}{n^2 \cdot 4^n}) x^n$

22. For $n = 0, 1, 2, 3, \dots$, let $a_n = \lceil \frac{4+2(-1)^n}{5} \rceil^n$.

(a) Find $\limsup (a_n)^{1/n}$, $\liminf (a_n)^{1/n}$, $\limsup |\frac{a_{n+1}}{a_n}|$ and $\liminf |\frac{a_{n+1}}{a_n}|$.

(b) Do the series $\sum a_n$ and $\sum (-1)^n a_n$ converge? Explain briefly.

23. Let $f_n(x) = \frac{1+2\cos^2 nx}{\sqrt{n}}$. Prove carefully that (f_n) converges uniformly to 0 on \mathbb{R} .

24. Prove that if $f_n \rightarrow f$ uniformly on a set S , and if $g_n \rightarrow g$ uniformly on S , then $f_n + g_n \rightarrow f + g$ uniformly on S .

25. Let $f_n(x) = \frac{x^n}{n}$. Show (f_n) is uniformly convergent on $[-1, 1]$ and specify the limit function.

26. Let $f_n(x) = \frac{n+\cos x}{2n+\sin^2 x}$ for all real numbers x .

(a) Show (f_n) converges uniformly on \mathbb{R} . Hint: First decide what the limit function is; then show (f_n) converges uniformly to it.

(b) Calculate $\lim_{n \rightarrow \infty} \int_2^7 f_n(x) dx$. Hint: Don't integrate f_n .

27. Show $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$ converges uniformly on \mathbb{R} to a continuous function.

28. Show $\sum_{n=1}^{\infty} \frac{x^n}{n^2 2^n}$ has radius of convergence 2 and the series converges uniformly to a continuous function on $[-2, 2]$.

29. (a) Show $\sum \frac{x^n}{1+x^n}$ converges for $x \in [0, 1)$

(b) Show that the series converges uniformly on $[0, a]$ for each a , $0 < a < 1$.

30. Suppose $\sum_{k=1}^{\infty} g_k$ and $\sum_{k=1}^{\infty} h_k$ converge uniformly on a set S . Show $\sum_{k=1}^{\infty} (g_k + h_k)$ converges uniformly on S .

UNIT-IV

31. Let $f(x) = x$ for rational x and $f(x) = 0$ for irrational x .

(a) Calculate the upper and lower Darboux integrals for f on the interval $[0, b]$.

(b) Is f integrable on $[0, b]$?

32. Let f be a bounded function on $[a, b]$. Suppose there exist sequences (U_n) and (L_n) of upper and lower Darboux sums for f such that $\lim(U_n - L_n) = 0$. Show f is integrable and $\int_a^b f = \lim U_n = \lim L_n$.

33. A function f on $[a, b]$ is called a step function if there exists a partition $P = \{a = u_0 < u_1 < \dots < u_m = b\}$ of $[a, b]$ such that f is constant on each interval (u_{j-1}, u_j) , say $f(x) = c_j$ for x in (u_{j-1}, u_j) .

(a) Show that a step function f is integrable and evaluate $\int_a^b f$.

(b) Evaluate the integral $\int_0^4 P(x) dx$ for the postage-stamp function.

34. Show $|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x) dx| \leq \frac{16\pi^3}{3}$.

35. Let f be a bounded function on $[a, b]$, so that there exists $B > 0$ such that $|f(x)| \leq B$ for all $x \in [a, b]$.

(a) Show

$$U(f^2, P) - L(f^2, P) \leq 2B[U(f, P) - L(f, P)]$$

for all partitions P of $[a, b]$. Hint: $f(x)^2 - f(y)^2 = [f(x) + f(y)][f(x) - f(y)]$

(b) Show that if f is integrable on $[a, b]$, then f^2 also is integrable on $[a, b]$.

36. Calculate

(a) $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x e^{t^2} dt$

(b) $\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt.$

37. Show that if f is a continuous real-valued function on $[a, b]$ satisfying $\int_a^b f(x)g(x)dx = 0$ for every continuous function g on $[a, b]$, then $f(x) = 0$ for all x in $[a, b]$.

**Skill Enhancement Course – I - FOR ALL SCIENCE FACULTY B.Sc., II
YEAR, III Semester
DEPARTMENTS**

COMPUTER BASICS AND AUTOMATION

Credits: 2

Theory: 2 hours/week

Marks - 50

Unit –I BASICS OF COMPUTERS

- 1.2 Introduction to computers- Computer parts and Characteristics of computer.
- 1.2. Generations of Computers, Classification of Computers, Basic computer organization.
- 1.3. Applications of Computer. Input and Output Devices- Input Devices, Output Devices.
- 1.4. Soft Copy Devices, Hard Copy Devices. Computer Memory and Processors.

Unit – II OFFICE AUTOMATION

- 1.1. Desktop - Word - Creation of files and folders, recycle Bin.
- 1.2. Web browser, Office Automation System, need for Office Automation System.
- 1.3. Excel – Tables, graphs
- 1.4. PowerPoint, Access to files and folders.

Text Book:

- 1. Reema Thareja “Fundamentals of Computers” Oxford University Press 2015.

References:

- 1. A. Goel, Computer Fundamentals, Pearson Education, 2010.
- 2. Spoken Tutorial on “Linux (Ubuntu), LibreOffice (Writer, Calc, Impress), Firefox”, as E-resource for Learning. <http://spoken-tutorial.org>

2.8 Algebra

DSC-1D

BS:404

Theory: 4 credits and Practicals: 1 credits
Theory: 4 hours /week and Practicals: 2 hours /week

Objective: The course is aimed at exposing the students to learn some basic algebraic structures like groups, rings etc.

Outcome: On successful completion of the course students will be able to recognize algebraic structures that arise in matrix algebra, linear algebra and will be able to apply the skills learnt in understanding various such subjects.

Unit- I

Groups: Definition and Examples of Groups- Elementary Properties of Groups-Finite Groups; Subgroups -Terminology and Notation -Subgroup Tests - Examples of Subgroups Cyclic Groups: Properties of Cyclic Groups - Classification of Subgroups Cyclic Groups-Permutation Groups: Definition and Notation -Cycle Notation-Properties of Permutations -A Check Digit Scheme Based on D_5 .

Unit- II

Isomorphisms ; Motivation- Definition and Examples -Cayley's Theorem Properties of Isomorphisms -Automorphisms-Cosets and Lagrange's Theorem Properties of Cosets 138 - Lagrange's Theorem and Consequences-An Application of Cosets to Permutation Groups -The Rotation Group of a Cube and a Soccer Ball -Normal Subgroups and Factor Groups ; Normal Subgroups-Factor Groups -Applications of Factor Groups -Group Homomorphisms - Definition and Examples -Properties of Homomorphisms -The First Isomorphism Theorem.

Unit- III

Introduction to Rings: Motivation and Definition -Examples of Rings -Properties of Rings -Subrings -Integral Domains : Definition and Examples -Characteristics of a Ring -Ideals and Factor Rings; Ideals -Factor Rings -Prime Ideals and Maximal Ideals.

Unit- IV

Ring Homomorphisms: Definition and Examples-Properties of Ring- Homomorphisms -The Field of Quotients Polynomial Rings: Notation and Terminology.

Text:

- Joseph A Gallian, *Contemporary Abstract algebra (9th edition)*

References:

- Bhattacharya, P.B Jain, S.K.; and Nagpaul, S.R, *Basic Abstract Algebra*
 - Fraleigh, J.B, *A First Course in Abstract Algebra.*
 - Herstein, I.N, *Topics in Algebra*
 - Robert B. Ash, *Basic Abstract Algebra*
 - I Martin Isaacs, *Finite Group Theory*
 - Joseph J Rotman, *Advanced Modern Algebra*
-

2.8.1 Practicals Question Bank

Algebra

Unit-I

1. Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.
2. Let G be a group with the property that for any x, y, z in the group, $xy = zx$ implies $y = z$. Prove that G is Abelian.
3. Prove that the set of all 3×3 matrices with real entries of the form

$$\begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

is a group under multiplication.

4. Let G be the group of polynomials under addition with coefficients from Z_{10} . Find the orders of $f(x) = 7x^2 + 5x + 4$, $g(x) = 4x^2 + 8x + 6$, and $f(x) + g(x)$
5. If a is an element of a group G and $|a| = 7$, show that a is the cube of some element of G .
6. Suppose that $\langle a \rangle$, $\langle b \rangle$ and $\langle c \rangle$ are cyclic groups of orders 6, 8, and 20, respectively. Find all generators of $\langle a \rangle$, $\langle b \rangle$, and $\langle c \rangle$.
7. How many subgroups does Z_{20} have? List a generator for each of these subgroups.
8. Consider the set $\{4, 8, 12, 16\}$. Show that this set is a group under multiplication modulo 20 by constructing its Cayley table. What is the identity element? Is the group cyclic? If so, find all of its generators.
9. Prove that a group of order 4 cannot have a subgroup of order 3.
10. Determine whether the following permutations are even or odd.
 - a. (135)
 - b. (1356)
 - c. (13567)
 - d. (12)(134)(152)
 - e. (1243)(3521).

Unit-II

11. Show that the mapping $a \rightarrow \log_{10} a$ is an isomorphism from R^+ under multiplication to R under addition.
12. Show that the mapping $f(a + bi) = a - bi$ is an automorphism of the group of complex numbers under addition.
13. Find all of the left cosets of $\{1, 11\}$ in $U(30)$.

14. Let C^* be the group of nonzero complex numbers under multiplication and let $H = \{a + bi \in C^* / a^2 + b^2 = 1\}$. Give a geometric description of the coset $(3 + 4i)H$. Give a geometric description of the coset $(c + di)H$.
15. Let $H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} / a, b, d \in R, ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2, R)$?
16. What is the order of the factor group $\frac{Z_{60}}{\langle 5 \rangle}$?
17. Let $G = U(16)$, $H = \{1, 15\}$, and $K = \{1, 9\}$. Are H and K isomorphic? Are G/H and G/K isomorphic?
18. Prove that the mapping from R under addition to $GL(2, R)$ that takes x to

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

is a group homomorphism. What is the kernel of the homomorphism?

19. Suppose that f is a homomorphism from Z_{30} to Z_{30} and $\text{Ker } f = \{0, 10, 20\}$. If $f(23) = 9$, determine all elements that map to 9.
20. How many Abelian groups (up to isomorphism) are there
- of order 6?
 - of order 15?
 - of order 42?
 - of order pq , where p and q are distinct primes?
 - of order pqr , where p, q , and r are distinct primes?

Unit-III

21. Let $M_2(Z)$ be the ring of all 2×2 matrices over the integers and let $R = \left\{ \begin{pmatrix} a & a \\ b & b \end{pmatrix} / a, b \in Z \right\}$. Prove or disprove that R is a subring of $M_2(Z)$.
22. Suppose that a and b belong to a commutative ring R with unity. If a is a unit of R and $b^2 = 0$, show that $a + b$ is a unit of R .
23. Let n be an integer greater than 1. In a ring in which $x^n = x$ for all x , show that $ab = 0$ implies $ba = 0$.
24. List all zero-divisors in Z_{20} . Can you see a relationship between the zero-divisors of Z_{20} and the units of Z_{20} ?
25. Let a belong to a ring R with unity and suppose that $a^n = 0$ for some positive integer n . (Such an element is called nilpotent.) Prove that $1 - a$ has a multiplicative inverse in R .
26. Let d be an integer. Prove that $Z[\sqrt{d}] = \{a + b\sqrt{d} / a, b \in Z\}$ is an integral domain.
27. Show that Z_n has a nonzero nilpotent element if and only if n is divisible by the square of some prime.

28. Find all units, zero-divisors, idempotents, and nilpotent elements in $Z_3 \oplus Z_6$.
29. Find all maximal ideals in
- Z_8 .
 - Z_{10} .
 - Z_{12} .
 - Z_n .
30. Show that $R[x]/\langle x^2 + 1 \rangle$ is a field.

Unit-IV

31. Prove that every ring homomorphism f from Z_n to itself has the form $f(x) = ax$, where $a^2 = a$.
32. Prove that a ring homomorphism carries an idempotent to an idempotent.
33. In Z , let $A = \langle 2 \rangle$ and $B = \langle 8 \rangle$. Show that the group A/B is isomorphic to the group Z_4 but that the ring A/B is not ring-isomorphic to the ring Z_4 .
34. Show that the number 9, 897, 654, 527, 609, 805 is divisible by 99.
35. Show that no integer of the form $111, 111, 111, \dots, 111$ is prime.
36. Let $f(x) = 4x^3 + 2x^2 + x + 3$ and $g(x) = 3x^4 + 3x^3 + 3x^2 + x + 4$, where $f(x), g(x) \in Z_5[x]$. Compute $f(x) + g(x)$ and $f(x).g(x)$.
37. Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ in $Z_7[x]$. Determine the quotient and remainder upon dividing $f(x)$ by $g(x)$.
38. Let $f(x)$ belong to $Z_p[x]$. Prove that if $f(b) = 0$, then $f(b^p) = 0$.
39. Is the mapping from Z_{10} to Z_{10} given by $x \rightarrow 2x$ a ring homomorphism?
40. Determine all ring homomorphisms from Z to Z .

**Skill Enhancement Course - II- B.Sc., II YEAR, IV Semester
FOR ALL SCIENCE FACULTY DEPARTMENTS
MULTIMEDIA AND APPLICATIONS**

**Credits: 2 Theory: 2 hours/week
Marks - 50**

Unit - I FONTS AND IMAGES

- 1.1.Multimedia: Introduction to multimedia, components, uses of multimedia, Multimedia applications, virtual reality.
- 1.2.Text: Fonts and Faces, Using Text in Multimedia, Font Editing and Design Tools, Hypermedia & Hypertext.
- 1.3.Images: Still Images – bitmaps, vector drawing, 3D drawing and rendering, natural, light and colors, computerized colors, color palettes, image file formats.

Unit – II AUDIO AND VIDEO

- 2.1.Sound: Digital Audio, MIDI Audio, MIDI vs Digital Audio, Audio File Formats.
- 2.2Video: How video works, analog video, digital video, video file formats, video shooting and editing.
- 2.3Animation: Principle of animations, animation techniques, animation file formats.

References:

- 1. Tay Vaughan, —Multimedia: Making it work, TMH, Eighth edition.2011
- 2. Ralf Steinmetz and KlaraNaharstedt, —Multimedia: Computing, Communications Applications, Pearson.2012
- 3. Keyes, —Multimedia Handbook, TMH,2000.
- 4. K. Andleigh and K. Thakkar, —Multimedia System Design, PHI.2013

KAKATIYA UNIVERSITY
B.Sc. PROGRAMME
Under CBCS System wef A.Y: 2020-21
Second Year : : Semester- III

BS-302 / SEC-2: BIO STATISTICS

[2 HPW, #Credits: 2, Marks: 50 (Internal:10, External:40)]

Unit-I

Descriptive and Relational Statistics: Data collection and tabulation, Graphical representation of data, Measures of central tendency (Mean, Median and Mode) with simple applications, Measures of dispersion (Range, Quartile Deviation, Mean Deviation, Standard Deviation, Standard error and Coefficient of variation) with simple applications, Concept of Skewness and Kurtosis.

Concept of correlation, computation of Karl-Pearson correlation coefficient, Spearman' s rank correlation coefficient and Simple linear regression with simple applications,

Unit-II

Probability and Inferential Statistics: Basic concepts and Basic terms of probability, Mathematical, Statistical and Axiomatic definitions of probability Conditional probability and independence of events, Addition and multiplication theorems (Statements only) with simple applications. Statements and applications of Binomial, Poisson and Normal distributions.

Concepts of Population, Sample, Parameter, Statistic, Null and Alternative hypotheses, Critical region, two types of errors, Level of significance. Tests of significance based on goodness of fit, means, variances using χ^2 test, t-test, F-test and analysis of variance (ANOVA).

References:

1. Irfan Ali Khan and Atiya Khanum: Fundamentals of Bio Statistics, Ukaaz Publications, HYD.
2. V. K. Kapoor and S. C. Gupta: Fundamentals of Mathematical Statistics, Sultan Chand & Sons, New Delhi.
3. V. K. Kapoor and S. C. Gupta: Statistical Methods, Sultan Chand & Sons, New Delhi.

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B.Sc. Under CBCS System wef A.Y: 2021-22

Third Year : : Semester - V

GENERIC ELECTIVE (Common to all students)

WATER RESOURCES MANAGEMENT

(4 hrs/week) (Taught by ant Science Dept) (Credits:4) (Marks:100)

UNIT-I:

Introduction to water resources management, different types of water resources, water resources and its importance, Global distribution of water. Hydrological cycle, Conservation of water, recycling of water.

Unit-II:

Rain water harvesting, methods of roof top rain water harvesting in urban setting: Direct method - Storing rain water in tanks for direct use; indirect methods - Recharge pits, bore wells/dug wells, Recharge trenches. Over use of surface and ground water and control measures.

UNIT-III:

Importance of water shed and water shed management, Rain water harvesting in rural setting: Check dams, percolation tanks, gabion structure, continuous contour trenches, staggered contour trenches, farm ponds. Surface water and ground water pollution, control measures.

UNIT-IV :

Mission Bhagiratha: Telangana government water grid project for drinking water supply - aims and objectives and method of implementation. Mission Kakatiya: Telangana government project for the restoration of minor irrigation tanks, aims and objectives and method of implementation.

Text books:

- 1) Water Resources, Conservation and Management by Chatterjee, S.N.
- 2) Groundwater hydrology by Todd
- 3) Watershed management by J.V.S.Murthy
- 4) Applied Hydrogeology by Fetter.

SEMESTER-V

Linear Algebra

(w.e.f. academic year 2019-20 batch onwards)

DSC-V

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: The students are exposed to various concepts like vector spaces, bases, dimension, Eigen values etc.

Outcome: After completion this course students appreciate its interdisciplinary nature.

Unit- I

Vector Spaces: Vector Spaces and Subspaces -Null Spaces, Column Spaces, and Linear Transformations -Linearly Independent Sets; Bases -Coordinate Systems -The Dimension of a Vector Space

Unit- II

Rank-Change of Basis - Eigenvalues and Eigenvectors - The Characteristic Equation

Unit- III

Diagonalization: -Eigenvectors and Linear Transformations -Complex Eigenvalues - Applications to Differential Equations.

Unit- IV

Orthogonality and Least Squares : Inner Product, Length, and Orthogonality -Orthogonal Sets -Orthogonal Projections - The Gram-Schmidt Process.

Text:

David C Lay, Linear Algebra and its Applications 4e

References:

- 1] S Lang, Introduction to Linear Algebra
- 2] Gilbert Strang , Linear Algebra and its Applications
- 3] Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence; Linear Algebra
- 4] Kuldeep Singh; Linear Algebra.
- 5] Sheldon Axler; Linear Algebra Done Right

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SEMESTER-VI

(A) Numerical Analysis

(w.e.f. academic year 2019-20 batch onwards)

DSE-VI

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: Students will be made to understand some methods of numerical analysis.
Outcome: Students realize the importance of the subject in solving some problems of algebra and calculus.

Unit- I

Errors in Numerical Calculations - Solutions of Equations in One Variable: The Bisection Method - The Iteration Method - The Method of False Position-Newton's Method - Muller's Method - solution of Systems of Nonlinear Equations.

Unit- II

Interpolation and Polynomial Approximation: Interpolation - Finite Differences - Differences of Polynomials - Newton's formula for Interpolation - Gauss's central differences formulae - Stirling's and Bessel's formula - Lagrange's Interpolation Polynomial - Divided differences - Newton's General Interpolation formula - Inverse Interpolation.

Unit- III

Curve Fitting: Least Square Curve Fitting: Fitting a Straight Line-Nonlinear Curve Fitting.
Numerical Differentiation and Integration: Numerical Differentiation - Numerical Integration: Trapezoidal Rule-Simpson's 1/3rd-Rule and Simpson's 3/8th-Rule - Boole's and Weddle's Rule - Newton's Cotes Integration Formulae.

Unit- IV

Numerical Solutions of Ordinary Differential Equations: Taylor's Series Method - Picard's Method - Euler's Methods - Runge Kutta Methods.

Text:

S.S.Sastry, Introductory Methods of Numerical Analysis, PHI

References:

- 1] Richard L. Burden and J. Douglas Faires, Numerical Analysis (9e)
- 2] M K Jain, S R K Iyengar and R K Jain, Numerical Methods for Scientific and Engineering computation
- 3] B. Bradie , A Friendly introduction to Numerical Analysis




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SEMESTER-VI

(B) Integral Transforms

(w.e.f. academic year 2019-20 batch onwards)

DSE - VI

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: Students will be exposed to Integral Transforms. The students also learning the Applications of Laplace Transforms to Differential Equations which arises in Physics and Engineering Problems.

Outcome: Students apply their knowledge to solve some problems on special functions and Differential Equations by using the Integral Transforms.

Unit-I

Laplace Transforms-Definition-Existence theorem-Laplace transforms of derivatives and integrals Periodic functions and some special functions.

Unit- II

Inverse Transformations - Convolution theorem - Heaviside's expansion formula.

Unit- III

Applications to ordinary Differential equations - solutions of simultaneous ordinary Differential equations - Applications to Partial Differential equations.

Unit- IV

Fourier Transforms- Sine and cosine transforms-Inverse Fourier Transforms.

Text:

Vasishtha and Gupta, Integral Transforms, Krishna Prakashan Media(P), Ltd, Meerut (2e)

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SEMESTER-VI

(C) Analytical Solid Geometry

(w.e.f. academic year 2019-20 batch onwards)

DSE - VI

Theory: 5 credits and Tutorials: 0 credits
Theory: 5 hours /week and Tutorials: 1 hours /week

Objective: Students learn to describe some of the surfaces by using analytical geometry.

Outcome: Students understand the beautiful interplay between algebra and geometry.

Unit- I

Sphere: Definition-The Sphere Through Four Given Points-Equations of a Circle- Intersection of a Sphere and a Line-Equation of a Tangent Plane-Angle of Intersection of Two Spheres-Radical Plane.

Unit- II

Cones and Cylinders: Definition-Condition that the General Equation of second degree Represents a Cone-Cone and a Plane through its Vertex -Intersection of a Line with a Cone.

Unit- III

The Right Circular Cone-The Cylinder- The Right Circular Cylinder.

Unit- IV

The Conicoid: The General Equation of the Second Degree-Intersection of Line with a Conicoid-Plane of contact-Enveloping Cone and Cylinder.

Text:

Shanti Narayan and P K Mittal, Analytical Solid Geometry (17e)

References:

- 1] Khaleel Ahmed, Analytical Solid Geometry
- 2] S L Loney , Solid Geometry
- 3] Smith and Minton, Calculus

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